DEEPAWALI ASSIGNMENT

CLASS 11 FOR TARGET IIT JEE 2012

SOLUTION

IMAGE OF SHRI GANESH LAXMI SARASWATI

Director & H.O.D. IITJEE Mathematics
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Wishing You & Your Family A Very Happy & Prosperous Deepawali

Time Limit : 6 Sitting Each of 75 Minutes duration approx.

NOTE: This assignment will be discussed on the very first day after Deepawali Vacation, hence come prepared.

PRACTICE TEST # 1

M.M. 80 ***Time : 75 Min.***

[STRAIGHT OBJECTIVE TYPE] **[9 × 3 = 27]**

Q.1 If $\log(x + z) + \log(x - 2y + z) = 2\log(x - z)$ then x, y, z are in
(A) A.P. (B) G.P. (C*) H.P. (D) A.G.P.

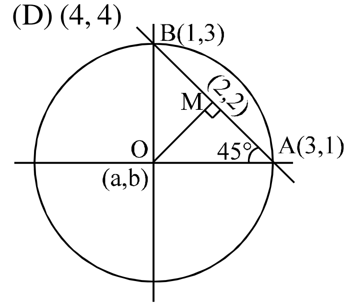
[Sol. $\log[(x + z)(x - 2y + z)] = 2\log(x - z) = \log(x - z)^2$
$\Rightarrow \quad (x + z)(x - 2y + z) = (x - z)^2 \quad \Rightarrow \quad (x + z)^2 - (x - z)^2 = 2y(x + z)$
$\Rightarrow \quad 4xz = 2y(x + z) \quad \Rightarrow \quad y = \dfrac{2xz}{x + z}$]

Q.2 If $x \in R$ and $b < c$, then $\dfrac{x^2 - bc}{2x - b - c}$ has no values.
(A) in $(-\infty, b)$ (B) in (c, ∞) (C*) between b and c (D) between – c and – b

[Sol. $y = \dfrac{x^2 - bc}{2x - b - c} \quad \Rightarrow \quad x^2 - 2yx + (b + c)y - bc = 0$
$\Delta \ge 0 \quad \Rightarrow \quad 4y^2 - 4(b + c)y + 4bc \ge 0$
$\Rightarrow \quad (y - b)(y - c) \ge 0 \quad \Rightarrow \quad y \in (-\infty, b] \cup [c, \infty)$]

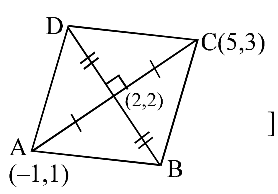
Q.3 The ends of a quadrant of a circle have the coordinates (1, 3) and (3, 1) then the centre of the such a circle is
(A*) (1, 1) (B) (2, 2) (C) (2, 6) (D) (4, 4)

[Hint: $(AM)^2 + (OM)^2 = (OA)^2$
$2 + (a - 2)^2 + (b - 2)^2 = (a - 3)^2 + (b - 1)^2$
$2 - 4a - 4b + 8 = -6a - 2b + 10$
$\Rightarrow \quad a = b$
Also $(OA)^2 + (OB)^2 = (AB)^2$
$2[(a - 1)^2 + (a - 3)^2] = 8$
$\Rightarrow \quad a = 1 \quad \text{or} \quad a = 3$]



Q.4 ABCD is a rhombus. If A is (–1, 1) and C is (5, 3), the equation of BD is
(A) $2x - 3y + 4 = 0$ (B) $2x - y + 3 = 0$ (C*) $3x + y - 8 = 0$ (D) $x + 2y - 1 = 0$

[Sol. Find equation of straight line through (2, 2) having slope – 3



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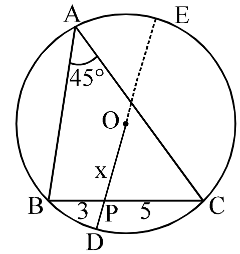
Q.5 Let ABC be a triangle with $\angle A = 45°$. Let P be a point on the side BC with PB = 3 and PC = 5. If 'O' is the circumcentre of the triangle ABC then the length OP is equal to
(A) $\sqrt{15}$ (B*) $\sqrt{17}$ (C) $\sqrt{18}$ (D) $\sqrt{19}$

[Sol. Using sine law

$$\frac{a}{\sin A} = 2R$$

$$8\sqrt{2} = 2R \quad \Rightarrow \quad R = 4\sqrt{2}$$



using power of a point

$$(PB)(PC) = (PD)(PE)$$

$$15 = (R - x)(R + x)$$

$$15 = R^2 - x^2 \quad \Rightarrow \quad x^2 = R^2 - 15 = 32 - 15 = 17$$

$\therefore$ $x = \sqrt{17}$ **Ans.**]

Q.6 If the sides of a right angled triangle are in A.P., then $\frac{R}{r}$ =

(A*) $\frac{5}{2}$ (B) $\frac{7}{3}$ (C) $\frac{9}{4}$ (D) $\frac{8}{3}$

[Sol. Let the sides be $a - d$, a, $a + d$

$(a - d)^2 + a^2 = (a + d)^2 \quad \Rightarrow \quad a = 4d$

The sides 3d, 4d, 5d

$$R = \frac{5d}{2}, \quad r = \frac{\Delta}{s} = \frac{6d^2}{6d} = d$$

$\therefore$ $\frac{R}{r} = \frac{5}{2}$ **Ans.**]

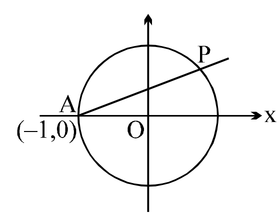
Q.7 Let C be a circle $x^2 + y^2 = 1$. The line *l* intersects C at the point (–1, 0) and the point P. Suppose that the slope of the line *l* is a rational number *m*. Number of choices for *m* for which both the coordinates of P are rational, is

(A) 3 (B) 4 (C) 5 (D*) infinitely many

[Sol. Equation of the line *l* is

$y - 0 = m(x + 1)$ ....(1)

solving it with $x^2 + y^2 = 1$



$$x^2 + m^2(x + 1)^2 = 1$$

$$(m^2 + 1)x^2 + 2m^2x + (m^2 - 1) = 0, \quad m \in Q$$

$$x = \frac{-2m^2 \pm \sqrt{4m^4 - 4(m^4 - 1)}}{2(m^2 + 1)} = \frac{-2m^2 \pm 2}{2(m^2 + 1)}$$

taking – ve sign $x = -1$ (corresponding to A)

with + ve sign $x = \frac{1 - m^2}{1 + m^2}$

since $m \in Q$ hence x will be rational.

If x is rational then y is also rational from (1)]

Q.8 One side of a rectangle lies along the line $4x + 7y + 5 = 0$, two of its vertices are (–3, 1) and (1, 1). Which of the following may be an equation of one of the other three straight lines?

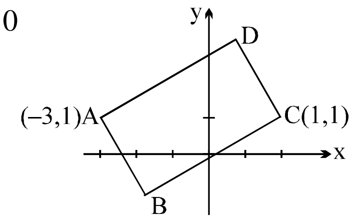
(A*) $7x - 4y = 3$ (B) $7x - 4y + 3 = 0$ (C) $y + 1 = 0$ (D) $4x + 7y = 3$

[Sol. Equation of line perpendicular to AD is A(–3, 1) lies on $4x + 7y + 5 = 0$

$7x - 4y = \lambda$.

It passes through (1, 1)

$\Rightarrow \quad \lambda = 3 \quad \Rightarrow \quad$ (A)]



Q.9 Three concentric circles of which the biggest is $x^2 + y^2 = 1$, have their radii in A.P. If the line $y = x + 1$ cuts all the circles in real and distinct points. The interval in which the common difference of the A.P. will lie is

(A) $\left(0, \frac{1}{4}\right)$ (B) $\left(0, \frac{1}{2\sqrt{2}}\right)$ (C*) $\left(0, \frac{2-\sqrt{2}}{4}\right)$ (D) none

[Sol. Radius of circle are r_1, r_2 and 1

line $y = x + 1$

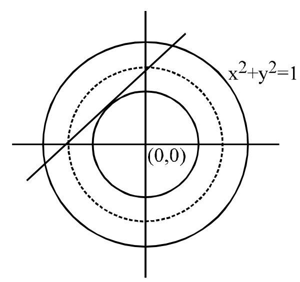
perpendicular from (0, 0) on line $y = x + 1$

$= \frac{1}{\sqrt{2}}$

now $r_1 > \frac{1}{\sqrt{2}}$ but $r_1 = 1 - 2d$

hence $1 - 2d > \frac{1}{\sqrt{2}}$; $\frac{\sqrt{2}-1}{\sqrt{2}} > 2d$; $d < \frac{\sqrt{2}-1}{2\sqrt{2}}$

$\therefore \quad d = \frac{\sqrt{2}-1}{2\sqrt{2}}$



Aliter : Equation of circle are

$x^2 + y^2 = 1$; $x^2 + y^2 = (1 - d)^2$; $x^2 + y^2 = (1 - 2d)^2$

$\Rightarrow$ solve any of circle with line $y = x + 1$

e.g. $x^2 + y^2 = (1 - d)^2 \Rightarrow 2x^2 + 2x + 2d - d^2 = 0$ cuts the circle in real and distinct point hence $\Delta > 0$

$\Rightarrow \quad 2d^2 - 4d + 1 > 0 \quad \Rightarrow \quad d = \frac{2 \pm \sqrt{2}}{4}$]

[COMPREHENSION TYPE] [3 × 3 = 9]

Paragraph for question nos. 10 to 12

Let A, B, C be three sets of real numbers (x, y) defined as

A : {(x, y): $y \ge 1$}

B : {(x, y): $x^2 + y^2 - 4x - 2y - 4 = 0$}

C : {(x, y): $x + y = \sqrt{2}$ }

Q.10 Number of elements in the $A \cap B \cap C$ is

(A) 0 (B*) 1 (C) 2 (D) infinite

Q.11 $(x + 1)^2 + (y - 1)^2 + (x - 5)^2 + (y - 1)^2$ has the value equal to

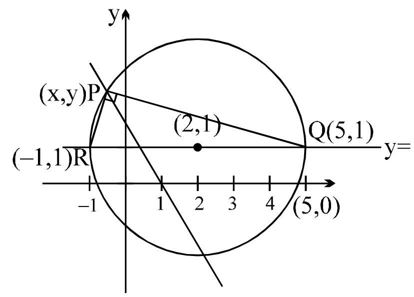
(A) 16 (B) 25 (C*) 36 (D) 49

Q.12 If the locus of the point of intersection of the pair of perpendicular tangents to the circle B is the curve S then the area enclosed between B and S is

(A) 6π (B) 8π (C*) 9π (D) 18π

[Sol.

(i) refer figure



(ii) when $y = 1$

$x^2 - 4x - 5 = 0$

$(x - 5)(x + 1) = 0$

$x = -1$ or $x = 5$

$(x + 1)^2 + (y - 1)^2 + (x - 5)^2 + (y - 1)^2 = (QR)^2 = 36$ Ans.

(iii) equation of director circle is

$$(x - 2)^2 + (y - 1)^2 = \left(3\sqrt{2}\right)^2 = 18$$

Area $= \pi\left[r_1^2 - r_2^2\right] = \pi[18 - 9] = 9\pi$ **Ans.**]

[MULTIPLE OBJECTIVE TYPE] **[2 × 4 = 8]**

Q.13 A circle passes through the points $(-1, 1)$, $(0, 6)$ and $(5, 5)$. The point(s) on this circle, the tangent(s) at which is/are parallel to the straight line joining the origin to its centre is/are :

(A) $(1, -5)$ (B*) $(5, 1)$ (C) $(-5, -1)$ (D*) $(-1, 5)$

[Hint : Note that Δ is right angled at $(0, 6)$. Centre of the circle is $(2, 3)$. Slope of the line joining origin to the centre is 3/2. Take parametric equation of a line through (2, 3) with

$\tan\theta = -\dfrac{2}{3}$ as $\dfrac{x-2}{\cos\theta} = \dfrac{y-3}{\sin\theta} = \pm r$ where $r = \sqrt{13}$.

Get the co–ordinates on the circle]

Q.14 If $al^2 - bm^2 + 2\,dl + 1 = 0$, where a, b, d are fixed real numbers such that $a + b = d^2$ then the line $lx + my + 1 = 0$ touches a fixed circle :

(A*) which cuts the x–axis orthogonally

(B) with radius equal to b

(C*) on which the length of the tangent from the origin is $\sqrt{d^2 - b}$

(D) none of these .

[Hint : $(d^2 - b)\,l^2 + 2\,dl + 1 = bm^2 \Rightarrow d^2l^2 + 2\,dl + 1 = b\,(l^2 + m^2)$

$\Rightarrow \left|\dfrac{d\ell + 1}{\sqrt{\ell^2 + m^2}}\right| = \left(\sqrt{b}\right)^2 \Rightarrow$ centre (d, 0) and radius b $\Rightarrow (x - d)^2 + y^2 = \left(\sqrt{b}\right)^2$]

[MATCH THE COLUMN] **[(3+3+3+3)×2=24]**

Q.15

	Column-I	Column-II
(A)	The equation $x^{x\sqrt{x}} = \left(x\sqrt{x}\right)^x$ has two solutions in positive real numbers x. One obvious solution is x = 1. The other one is x =	(P) 8/3
(B)	Suppose a triangle ABC is inscribed in a circle of radius 10 cm. If the perimeter of the triangle is 32 cm then the value of $\sin A + \sin B + \sin C$ equals	(Q) 9/4 (R) 5/4
(C)	Sum of infinte terms of the series $1 + \dfrac{3}{4} + \dfrac{7}{16} + \dfrac{15}{64} + \dfrac{31}{256} + ...$ equals	(S) 8/5
(D)	The sum of $\displaystyle\sum_{r=1}^{\infty}\left(\frac{r+3}{r(r+1)(r+2)}\right)$ equals	

[Ans. (A) Q; (B) S; (C) P; (D) R]

[Sol. (A) Take log on both the sides.

(B) Given $a + b + c = 32$; $R = 30$ cm

$$\sum \sin A = \frac{a+b+c}{2R} \quad \text{(using sine law)}$$

$$= \frac{32}{20} = \frac{8}{5} \text{ Ans.]}$$

Q.16

	Column-I		Column-II
(A)	If the line $x + 2ay + a = 0$, $x + 3by + b = 0$ & $x + 4cy + c = 0$ are concurrent, then a, b, c are in	(P)	A.P.
(B)	The points with the co-ordinates $(2a, 3a)$, $(3b, 2b)$ & (c, c) are collinear then a, b, c are in	(Q)	G.P.
(C)	If the lines, $ax + 2y + 1 = 0$; $bx + 3y + 1 = 0$ & $cx + 4y + 1 = 0$ passes through the same point then a, b, c are in	(R)	H.P.
(D)	Let a, b, c be distinct non–negative numbers. If the lines $ax + ay + c = 0$, $x + 1 = 0$ & $cx + cy + b=0$ pass through the same point then a, b, c are in	(S)	neither A.P. nor G.P. nor H.P.

[Ans. (A) R; (B) S; (C) P; (D) Q]

[Sol.**(B)** $\begin{vmatrix} 2a & 3a & 1 \\ 3b & 2b & 1 \\ c & c & 1 \end{vmatrix} = 0$ solving it we get, $2a(2b - c) - 3a(3b - c) + 1(3bc - 2bc) = 0$

$4ab - 2ac - 9ab + 3ac + 3bc - 2bc = 0$

$-5ab + ac + bc = 0$

or $\frac{1}{a} + \frac{1}{b} = \frac{5}{c}$ or $\frac{2ab}{a+b} = \frac{2c}{5}$ $\Rightarrow$ a, $\frac{2c}{5}$, b in H.P.]

[SUBJECTIVE TYPE]

Q.17 Find the sum of the series $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + + \frac{1^3+2^3+3^3}{1+3+5} + ...$ upto 16 terms. **[6]**

[Ans. 446]

[Sol. The r^{th} term, $t_r = \frac{1^3+2^3+3^3+.......+r^3}{1+3+5+.......+(2r-1)} = \left(\frac{r(r+1)}{2}\right)^2 \frac{1}{r^2} = \frac{1}{4}(r+1)^2$

$$\sum_{r=1}^{16} t_r = \frac{1}{4}[2^2+3^2+......+17^2] = \frac{1}{4}\left[\frac{17\times18\times35}{6} - 1\right] = 446 \textbf{ Ans.]}$$

Q.18 Find the number of circles that touch all the three lines $2x - y = 5$, $x + y = 3$, $4x - 2y = 7$. **[6]**

[Ans. 4]

PRACTICE TEST # 2

M.M. 80 ***Time : 75 min.***

[STRAIGHT OBJECTIVE TYPE] [8 × 3 = 24]

Q.1 If the sum of m consecutive odd integers is m^4, then the first integer is
(A) $m^3 + m + 1$ (B) $m^3 + m - 1$ (C) $m^3 - m - 1$ (D*) $m^3 - m + 1$

[Sol. Let $2a + 1, 2a + 3, 2a + 5$, ... be the A.P.

Sum $= m^4 = \frac{m}{2}[2(2a + 1) + (m - 1)2 = m^4 \quad \Rightarrow \quad 2a + m = m^3 \; ; \; 2a + 1 = m^3 - m + 1$ **Ans.**]

Q.2 The values of x for which the inequalities $x^2 + 6x - 27 > 0$ and $-x^2 + 3x + 4 > 0$ hold simultaneously lie in
(A) $(-1, 4)$ (B) $(-\infty, -9) \cup (3, \infty)$
(C) $(-9, -1)$ (D*) $(3, 4)$

[Sol. $x^2 + 6x - 27 > 0 \Rightarrow (x - 3)(x + 9) > 0 \Rightarrow x \in (-\infty, -9) \cup (3, \infty)$ ....(1)
$-x^2 + 3x + 4 > 0 \Rightarrow x^2 - 3x - 4 < 0 \Rightarrow (x - 4)(x + 1) < 0 \Rightarrow x \in (-1, 4)$
The intersection of two sets in (1), (2) is (3, 4) **Ans.**]

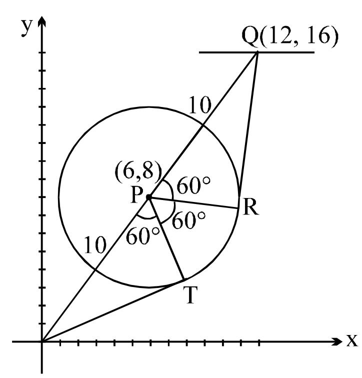
Q.3 The diagonals of the quadrilateral whose sides are $lx + my + n = 0$, $mx + ly + n = 0$, $lx + my + n_1 = 0$, $mx + ly + n_1 = 0$ include an angle

(A) $\frac{\pi}{4}$ (B*) $\frac{\pi}{2}$ (C) $\tan^{-1}\left(\frac{l^2 - m^2}{l^2 + m^2}\right)$ (D) $\tan^{-1}\left(\frac{2lm}{l^2 + m^2}\right)$

Q.4 In the xy-plane, the length of the shortest path from (0, 0) to (12, 16) that does not go inside the circle $(x - 6)^2 + (y - 8)^2 = 25$ is

(A) $10\sqrt{3}$ (B) $10\sqrt{5}$ (C*) $10\sqrt{3} + \frac{5\pi}{3}$ (D) $10 + 5\pi$

[Sol. Let O = (0, 0), P = (6, 8) and Q = (12, 16).
As shown in the figure the shortest route consists of tangent OT, minor arc TR and tangent RQ.
Since OP = 10, PT = 5, and $\angle OTP = 90°$,
it follows that $\angle OPT = 60°$ and OT = $5\sqrt{3}$.
By similar reasoning, $\angle QPR = 60°$ and QR = $5\sqrt{3}$.
Because O, P and Q are collinear (why?),
$\angle RPT = 60°$, so arc TR is of length $\frac{5\pi}{3}$.



Hence the length of the shortest route is $2(5\sqrt{3}) + \frac{5\pi}{3}$ Ans.]

Q.5 If $a_1, a_2, ..., a_n$ are in A.P. where $a_i > 0$ for all i,

then $\dfrac{1}{\sqrt{a_1}+\sqrt{a_2}} + \dfrac{1}{\sqrt{a_2}+\sqrt{a_3}} + ... + \dfrac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}$ equals

(A) $\dfrac{1}{\sqrt{a_1}+\sqrt{a_n}}$ (B) $\dfrac{n}{\sqrt{a_1}+\sqrt{a_n}}$ (C) $\dfrac{n+1}{\sqrt{a_1}+\sqrt{a_n}}$ (D*) $\dfrac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$

[Sol. Let d be the common difference

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + ... + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}$$

$$= \frac{\sqrt{a_2}-\sqrt{a_1}}{d} + \frac{\sqrt{a_3}-\sqrt{a_2}}{d} + ... + \frac{\sqrt{a_n}-\sqrt{a_{n-1}}}{d} = \frac{\sqrt{a_n}-\sqrt{a_1}}{d}$$, cancelling the terms

$$= \frac{a_n - a_1}{\left(\sqrt{a_n}+\sqrt{a_1}\right)d} = \frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$$ **Ans.**]

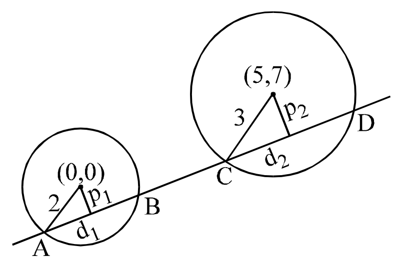
Q.6 The equation of a line inclined at an angle $\dfrac{\pi}{4}$ to the axis X, such that the two circles $x^2 + y^2 = 4$, $x^2 + y^2 - 10x - 14y + 65 = 0$ intercept equal lengths on it, is

(A*) $2x - 2y - 3 = 0$ (B) $2x - 2y + 3 = 0$ (C) $x - y + 6 = 0$ (D) $x - y - 6 = 0$

[Sol. Let equation of line be $y = x + c$

$y - x = c$ ....(1)

perpendicular from (0, 0) on (1) is $\left|\dfrac{-c}{\sqrt{2}}\right| = \dfrac{c}{\sqrt{2}}$



In ΔAON, $\sqrt{2^2 - \left(\dfrac{c}{\sqrt{2}}\right)^2} = AN$

and in ΔCPM, $\sqrt{3^2 - 2 - \dfrac{c}{\sqrt{2}}} = CM$

perpendicular from (5, 7) on line $y - x = c = \dfrac{2-c}{\sqrt{2}}$

Given $AN = CM = 4 - \dfrac{c^2}{2} = 9 - \dfrac{(2-c)^2}{2} \Rightarrow c = -\dfrac{3}{2}$

$\therefore$ equation of line $y = x - \dfrac{3}{2}$ of $2x - 2y - 3 = 0$]

Q.7 If the straight line $y = mx$ is outside the circle $x^2 + y^2 - 20y + 90 = 0$, then

(A) $m > 3$ (B) $m < 3$ (C) $| m | > 3$ (D*) $| m | < 3$

[Sol. Centre (0,10), radius $\sqrt{10}$.

Distance of (0,10) from $y = mx$ is greater then $\sqrt{10}$ i.e. $\dfrac{10}{\sqrt{m^2+1}} > \sqrt{10} < 3$]

Q.8 A line with gradient 2 intersects a line with gradient 6 at the point (40, 30). The distance between x-intercepts of these lines, is
(A) 6 (B) 8 (C*) 10 (D) 12

[Sol. Let C_1 and C_2 be the x-intercept of lines with slope 2 and 6 respectively

$y - 0 = 2(x - c_1)$

$y = 2x - 2C_1$ ....(1)

|||ly $y = 6x - 6C_2$ ....(2)

both (1) and (2) satisfy x = 40 and y = 30

$30 = 80 - 2C_1 \Rightarrow C_1 = 25$

and $30 = 240 - 6C_2$

$\Rightarrow 6 \cdot C_2 = 210 \Rightarrow C_2 = 35$

hence $C_2 - C_1 = 10$ Ans.]

[COMPREHENSION TYPE] [3 × 3 = 9]

Paragraph for question nos. 9 to 11

Consider a circle $x^2 + y^2 = 4$ and a point P(4, 2). θ denotes the angle enclosed by the tangents from P on the circle and A, B are the points of contact of the tangents from P on the circle.

Q.9 The value of θ lies in the interval
(A) (0, 15°) (B) (15°, 30°) (C) 30°, 45°) (D*) (45°, 60°)

Q.10 The intercept made by a tangent on the x-axis is
(A) 9/4 (B*) 10/4 (C) 11/4 (D) 12/4

Q.11 Locus of the middle points of the portion of the tangent to the circle terminated by the coordinate axes is
(A*) $x^{-2} + y^{-2} = 1^{-2}$ (B) $x^{-2} + y^{-2} = 2^{-2}$ (C) $x^{-2} + y^{-2} = 3^{-2}$ (D) $x^{-2} - y^{-2} = 4^{-2}$

[Sol. Tangent

$y - 2 = m(x - 4)$

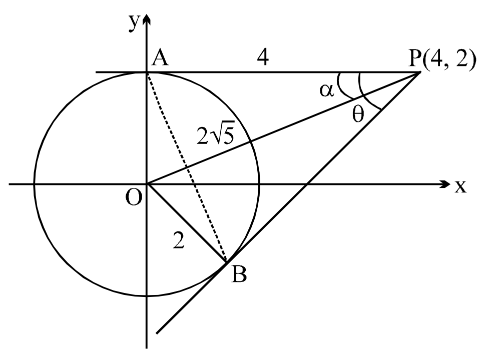
$mx - y + (2 - 4m) = 0$

$p = \left|\frac{2-4m}{\sqrt{1+m^2}}\right| = 2$

$(1 - 2m)^2 = 1 + m^2$

$3m^2 - 4m = 0$

$m = 0$ or $m = \frac{4}{3}$



Hence equation of tangent is y = 2 and (with infinite intercept on x-axis)

or $y - 2 = \frac{4}{3}(x - 4) \Rightarrow 3y - 6 = 4x - 16 \Rightarrow 4x - 3y - 10 = 0$

x-intercept $= \frac{10}{4}$ **Ans.(ii)** ⇒ **(B)**

Variable line with mid point (h, k)

$\frac{x}{2h} + \frac{y}{2k} = 1$, it touches the circle $x^2 + y^2 = 4$

$\therefore \left|\frac{-1}{\sqrt{\frac{1}{4h^2}+\frac{1}{4k^2}}}\right| = 2 \Rightarrow \frac{1}{4h^2} + \frac{1}{4k^2} = \frac{1}{4} \Rightarrow$ locus is $x^{-2} + y^{-2} = 1$ **Ans.(iii)** ⇒ **(A)**]

[REASONING TYPE] **[1 × 3 = 3]**

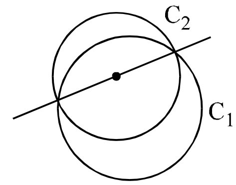
Q.12 Statement-1: The circle $C_1 : x^2 + y^2 - 6x - 4y + 9 = 0$ bisects the circumference of the circle $C_2 : x^2 + y^2 - 8x - 6y + 23 = 0$.

because

Statement-2: Centre of the circle C_1 lies on the circumference of C_2.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B*) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

[Sol. C_1 : centre (3, 2)
C_2 : centre (4, 3)
radical axis of C_1 and C_2 is



$C_1 - C_2 = 0$
$2x + 2y - 14 = 0$
$x + y - 7 = 0$ ....(1)

since (1) passes through the centre of C_2 (4, 3) hence S-1 is correct.
also (3, 2) lies on C_2 hence S-2 is correct but that is not the correct explanation of S-1.]

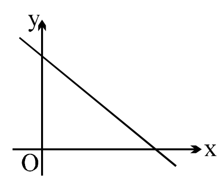
[MULTIPLE OBJECTIVE TYPE] **[2 × 4 = 8]**

Q.13 Which of the following lines have the intercepts of equal lengths on the circle,
$x^2 + y^2 - 2x + 4y = 0$?
(A*) $3x - y = 0$ (B*) $x + 3y = 0$ (C*) $x + 3y + 10 = 0$ (D*) $3x - y - 10 = 0$

[Hint : Chords equidistance from the centre are equal]

Q.14 Three distinct lines are drawn in a plane. Suppose there exist exactly *n* circles in the plane tangent to all the three lines, then the possible values of *n* is/are
(A*) 0 (B) 1 (C*) 2 (D*) 4

[Sol. Case-1: If lines form a triangle then n = 4
i.e. 3 excircles and 1 incircle
Case-2: If lines are concurrent
or all 3 parallel then n = 0
Case-3: If two are parallel
and third cuts then n = 2
hence (A), (C), (D)]



[MATCH THE COLUMN] **[(3+3+3+3)×2=24]**

Q.15 Consider the line $Ax + By + C = 0$.
Match the nature of intercept of the line given in **column-I** with their corresponding conditions in **column-II**. The mapping is one to one only.

	Column-I		**Column-II**
(A)	x intercept is finite and y intercept is infinite	(P)	$A = 0,\ B, C \neq 0$
(B)	x intercept is infinite and y intercept is finite	(Q)	$C = 0,\ A, B \neq 0$
(C)	both x and y intercepts are zero	(R)	$A, B = 0$ and $C \neq 0$
(D)	both x and y intercepts are infinite	(S)	$B = 0,\ A, C \neq 0$

[Ans. (A) S; (B) P; (C) Q; (D) R]

Q.16		Column I	Column II	
	(A)	If the lines $ax + 2y + 1 = 0$, $bx + 3y + 1 = 0$ and $cx + 4y + 1 = 0$ passes through the same point, then a, b, c are in	(P)	A.P.
	(B)	Let a, b, c be distinct non-negative numbers. If the lines $ax + ay + c = 0$, $x + 1 = 0$ and $cx + cy + b = 0$ passes through the same point, then a, b, c are in	(Q)	G.P.
	(C)	If the lines $ax + amy + 1 = 0$, $bx + (m + 1)by + 1 = 0$ and $cx + (m + 2)cy + 1 = 0$, where $m \neq 0$ are concurrent then a, b, c are in	(R)	H.P.
	(D)	If the roots of the equation $x^2 - 2(a + b)x + a(a + 2b + c) = 0$ be equal then a, b, c are in	(S)	None

[Ans. (A) P; (B) S; (C) R; (D) Q]

[Hint: (D) Roots equal $\Rightarrow$ D = 0

$\therefore$ $4(a + b)^2 = 4a(a + 2b + c)$

$a^2 + b^2 + 2ab = a^2 + 2ab + ac$

$\therefore$ $b^2 = ac$ $\Rightarrow$ a, b, c are in G.P. $\Rightarrow$ (Q)]

[SUBJECTIVE TYPE]

Q.17 If S_1, S_2, S_3 are the sum of n, 2n, 3n terms respectively of an A.P. then find the value of $\frac{S_3}{(S_2 - S_1)}$.

[6]

[Ans. 3]

[Sol. $S_1 = \frac{n}{2}[2a + (n - 1)d]$; $S_2 = n[2a + (2n - 1)d]$

$S_2 - S_1 = na + (3n - 1)\frac{nd}{2} = \frac{n}{2}[2a + (3n - 1)d]$

$S_3 = \frac{3n}{2}[2a + (3n - 1)d]$

$\therefore$ $\frac{S_3}{(S_2 - S_1)} = 3$ **Ans.**]

Q.18 Find the distance of the centre of the circle $x^2 + y^2 = 2x$ from the common chord of the circles $x^2 + y^2 + 5x - 8y + 1 = 0$ and $x^2 + y^2 - 3x + 7y + 25 = 0$. [Ans. 2] **[6]**

[Sol. The common chord is $8x - 15y + 26 = 0$

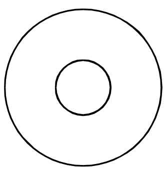
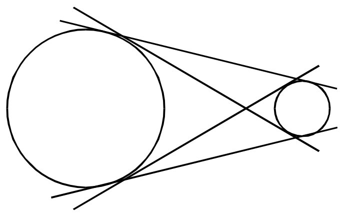
Distance of (1, 0) is $\frac{8 + 26}{\sqrt{8^2 + 15^2}} = \frac{34}{17} = 2$ **Ans.**]

PRACTICE TEST # 3

M.M. 68 ***Time : 75 Min.***

[STRAIGHT OBJECTIVE TYPE] **[10 × 3 = 30]**

Q.1 Suppose that two circles C_1 and C_2 in a plane have no points in common. Then
(A) there is no line tangent to both C_1 and C_2.
(B) there are exactly four lines tangent to both C_1 and C_2.
(C) there are no lines tangent to both C_1 and C_2 or there are exactly two lines tangent to both C_1 and C_2.
(D*) there are no lines tangent to both C_1 and C_2 or there are exactly four lines tangent to both C_1 and C_2.

[Hint:  or ]

Q.2 If cos(x – y), cos x, cos (x + y) are in H.P., then the value of $\cos x \sec\frac{y}{2}$ is

(A) ± 1 (B) $\pm\frac{1}{\sqrt{2}}$ (C*) $\pm\sqrt{2}$ (D) $\pm\sqrt{3}$

[Sol. cos (x – y), cos x, cos (x + y) are in H.P.

$$\therefore \quad \cos x = \frac{2\cos(x-y)\cos(x+y)}{\cos(x-y)+\cos(x+y)} = \frac{\cos^2 x - \sin^2 y}{\cos x \cos y}$$

$$\Rightarrow \quad \sin^2 y = \cos^2 x(1-\cos y) = 2\cos^2 x \sin^2\frac{y}{2}$$

$$\Rightarrow \quad 4\sin^2\frac{y}{2}\cos^2\frac{y}{2} = 2\cos^2 x\sin^2\frac{y}{2} \quad \Rightarrow \quad \cos^2 x = 2\cos^2\frac{y}{2} \quad \Rightarrow \quad \cos^2 x\sec^2\frac{y}{2} = 2$$

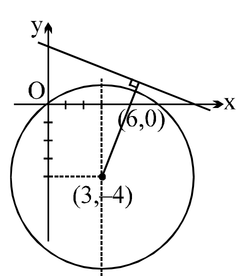
$$\Rightarrow \quad \cos x\sec\frac{y}{2} = \pm\sqrt{2}$$ **Ans.**]

Q.3 The shortest distance from the line $3x + 4y = 25$ to the circle $x^2 + y^2 = 6x - 8y$ is equal to
(A*) 7/5 (B) 9/5 (C) 11/5 (D) 32/5

[Sol. Centre: (3 , – 4) and r = 5
perpendicular distance from (3, – 4) on
$3x + 4y - 25 = 0$ is

$$p = \left|\frac{9-16-25}{5}\right| = \frac{32}{5}$$

$$d = \frac{32}{5} - 5 = \frac{7}{5}$$ **Ans.**]



Q.4 The expression $a(x^2 - y^2) - bxy$ admits of two linear factors for
(A) a + b = 0 (B) a = b (C) $4a = b^2$ (D*) all a and b.

[Sol. The expression $ax^2 + bxy + cy^2$ is the product of two linear factors if and only if the discriminant ≥ 0.
The discriminant of $ax^2 - bxy - ay^2$ is $b^2 + 4a^2 \geq 0$.
The discriminant of $ax^2 - bxy - ay^2$ is $b^2 + 4a^2 \geq 0$ for all a and b.]

Q.5 The points (x_1, y_1), (x_2, y_2), (x_1, y_2) and (x_2, y_1) are always :
(A) collinear (B*) concyclic
(C) vertices of a square (D) vertices of a rhombus
[Hint : All the points lie on the circle $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$]

Q.6 If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$
where a, b, c are in A.P. and $|a| < 1$, $|b| < 1$, $|c| < 1$, then x, y, z are in
(A) A.P. (B) G.P. (C*) H.P. (D) A.G.P.

[Sol. $x = 1 + a + a^2 + ... \infty \Rightarrow x = \frac{1}{1-a}$; |||ly $y = \frac{1}{1-b}$, $z = \frac{1}{1-c}$

$\Rightarrow \quad 1 - a = \frac{1}{x},\ 1 - b = \frac{1}{y},\ 1 - c = \frac{1}{z}$

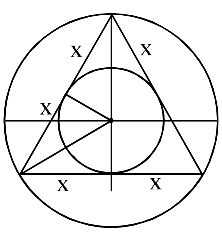
$a = 1 - \frac{1}{x},\ b = 1 - \frac{1}{y},\ c = 1 - \frac{1}{z}$

a, b, c are in A.P $\Rightarrow$ $1 - \frac{1}{x},\ 1 - \frac{1}{y},\ 1 - \frac{1}{z}$ are in A.P. $\Rightarrow$ $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

$\Rightarrow$ x, y, z are in H.P.]

Q.7 Tangents are drawn from any point on the circle $x^2 + y^2 = R^2$ to the circle $x^2 + y^2 = r^2$. If the line joining the points of intersection of these tangents with the first circle also touch the second, then R equals

(A) $\sqrt{2}\,r$ (B*) 2r (C) $\frac{2r}{2-\sqrt{3}}$ (D) $\frac{4r}{3-\sqrt{5}}$

[HInt: 

]

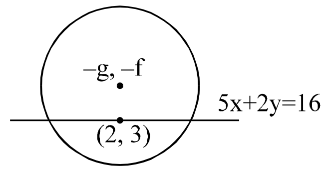
Q.8 The greatest slope along the graph represented by the equation $4x^2 - y^2 + 2y - 1 = 0$, is
(A) – 3 (B) – 2 (C*) 2 (D) 3
[Hint: $y^2 - 2y + 1 = 4x^2$
$(y - 1) = 2x$ or $-2x$
$y = 2x + 1$ or $y = 1 - 2x$
greatest slope = 2 Ans.]

Q.9 The locus of the center of the circles such that the point (2 , 3) is the mid point of the chord $5x + 2y = 16$ is

(A*) $2x - 5y + 11 = 0$ (B) $2x + 5y - 11 = 0$

(C) $2x + 5y + 11 = 0$ (D) none

[Hint : Slope of the given line = – 5/2



$$\Rightarrow -\frac{5}{2}\cdot\frac{3+f}{2+g} = -1 \Rightarrow 15 + 5f = 4 + 2g$$

$\Rightarrow$ locus is $2x - 5y + 11 = 0$]

Q.10 The number of distinct real values of λ, for which the determinant $\begin{vmatrix} -\lambda^2 & 1 & 1 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix}$ vanishes, is

(A) 0 (B) 1 (C*) 2 (D) 3

[Sol. $R_1 \to R_1 + R_2 + R_3$

$$(2 - \lambda^2)\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix} = 0$$

$C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$

$$(2 - \lambda^2)\begin{vmatrix} 0 & 0 & 1 \\ 1+\lambda^2 & -\lambda^2-1 & 1 \\ 0 & 1+\lambda^2 & -\lambda^2 \end{vmatrix} = 0 \Rightarrow (2 - \lambda^2)[1 + \lambda^2]^2 = 0$$

$\therefore$ $\lambda^2 = 2 \Rightarrow \lambda = \pm\sqrt{2} \Rightarrow$ two values of λ]

[COMPREHENSION TYPE] [3 × 3 = 9]

Paragraph for questions nos. 11 to 13

Consider the two quadratic polynomials

$$C_a : y = \frac{x^2}{4} - ax + a^2 + a - 2 \quad \text{and} \quad C : y = 2 - \frac{x^2}{4}$$

Q.11 If the origin lies between the zeroes of the polynomial C_a then the number of integral value(s) of 'a' is

(A) 1 (B*) 2 (C) 3 (D) more than 3

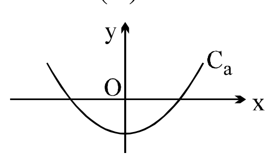
Q.12 If 'a' varies then the equation of the locus of the vertex of C_a, is

(A*) $x - 2y - 4 = 0$ (B) $2x - y - 4 = 0$ (C) $x - 2y + 4 = 0$ (D) $2x + y - 4 = 0$

Q.13 For a = 3, if the lines $y = m_1x + c_1$ and $y = m_2x + c_2$ are common tangents to the graph of C_a and C then the value of $(m_1 + m_2)$ is equal to

(A) – 6 (B*) – 3 (C) 1/2 (D) none

[Sol. $y = f(x) = \frac{x^2}{4} - ax + a^2 + a - 2$



(i) for zeroes to be on either side of origin

$f(0) < 0$

$a^2 + a - 2 < 0 \Rightarrow (a + 2)(a - 1) < 0 \Rightarrow -2 < a < 1 \Rightarrow$ 2 integers i.e. {–1, 0} $\Rightarrow$ **(B)**

(ii) Vertex of C_a is $(2a, a-2)$

hence $h = 2a$ and $k = a - 2$

$h = 2(k+2)$

locus $x = 2y + 4 \Rightarrow x - 2y - 4 = 0$ **Ans.**

(iii) Let $y = mx + c$ is a common tangent to $y = \frac{x^2}{4} - 3x + 10$ ....(1) (for a = 3)

and $y = 2 - \frac{x^2}{4}$ ....(2) where $m = m_1$ or m_2 and $c = c_1$ or c_2

solving $y = mx + c$ with (1)

$$mx + c = \frac{x^2}{4} - 3x + 10$$

or $\frac{x^2}{4} - (m+3)x + 10 - c = 0$

D = 0 gives

$(m+3)^2 = 10 - c \Rightarrow c = 10 - (m+3)^2$ ....(3)

|||ly $mx + c = 2 - \frac{x^2}{4} \Rightarrow \frac{x^2}{4} + mx + c - 2 = 0$

D = 0 gives

$m^2 = c - 2 \Rightarrow c = 2 + m^2$ ....(4)

from (3) and (4)

$10 - (m+3)^2 = 2 + m^2 \Rightarrow 2m^2 + 6m + 1 = 0$

$\Rightarrow m_1 + m_2 = -\frac{6}{2} = -3$ **Ans.**]

[REASONING TYPE] [1 × 3 = 3]

Q.14 Statement-1: Angle between the tangents drawn from the point P(13, 6) to the circle $S : x^2 + y^2 - 6x + 8y - 75 = 0$ is 90°.

because

Statement-2: Point P lies on the director circle of S.

(A*) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

[Hint: Equation of director's circle is $(x-3)^2 + (y+4)^2 = 200$ and point (13, 6) satisfies the given circle $(x-3)^2 + (y+4)^2 = 100$]

[MULTIPLE OBJECTIVE TYPE] [2 × 4 = 8]

Q.15 The fourth term of the A.G.P. 6, 8, 8, ..., is

(A*) 0 (B) 12 (C) $\frac{32}{3}$ (D*) $\frac{64}{9}$

[Sol. $6, (6+d)r, (6+2d)r^2, (6+3d)r^3$ are in A.G.P.

$(6+d)r = 8, (6+2d)r^2 = 8$

Eliminating r, $(6+d)^2 = 8(6+2d)$

$\Rightarrow d^2 - 4d - 12 = 0 \Rightarrow d = -2, 6$

$d = -2 \Rightarrow r = 2, t_4 = (6+3d)r^3 = 0$ **Ans.**

$d = 6 \quad \Rightarrow \qquad r = \frac{2}{3}$, $t_4 = 6 + 3d)r^3 = 24 \times \frac{8}{27} = \frac{64}{9}$ **Ans.**]

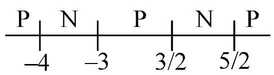
Q.16 $\frac{8x^2+16x-51}{(2x-3)(x+4)} > 3$ if

(A*) $x < -4$ (B*) $x > \frac{5}{2}$ (C) $-1 < x < 1$ (D*) $-3 < x < \frac{3}{2}$

[Sol. $\frac{8x^2+16x-51}{(2x-3)(x+4)} > 3 \quad \Rightarrow \quad \frac{2x^2+x-15}{(2x-3)(x+4)} = \frac{(x+3)(2x-5)}{(2x-3)(x+4)} > 0$

Multiplying by $(2x-3)^2(x+4)^2$,

$(x+3)(2x-5)(2x-3)(x+4)^2 > 0$



$\therefore \qquad x \in (-\infty, -4) \cup \left(-3, \frac{3}{2}\right) \cup \left(\frac{5}{2}, \infty\right)$]

[MATCH THE COLUMN] [3+3+3+3=12]

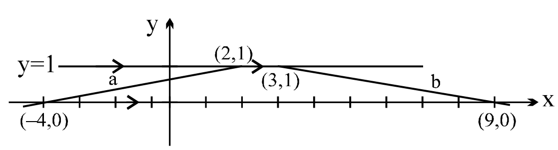
Q.17

	Column-I		Column-II
(A)	The lines $y = 0$; $y = 1$; $x - 6y + 4 = 0$ and $x + 6y - 9 = 0$ constitute a figure which is	(P)	a cyclic quadraliteral
(B)	The points A(a, 0), B(0, b), C(c, 0) and D(0, d) are such that $ac = bd$ and a, b, c, d are all non-zero. The points A, B, C and D always constitute	(Q)	a rhombus
(C)	The figure formed by the four lines $ax \pm by \pm c = 0$ $(a \neq b)$, is	(R)	a square
(D)	The line pairs $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$ constitute a figure which is	(S)	a trapezium

[Ans. (A) P, S; (B) P; (C) Q; (D) P, Q, R]

[Sol.

(A) obviously trapezium

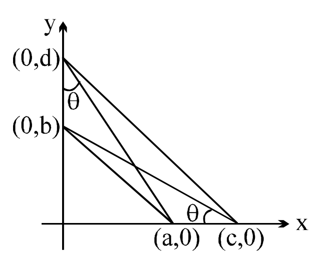


$\left.\begin{array}{l} a = \sqrt{37} \\ b = \sqrt{37} \end{array}\right] \quad \Rightarrow \quad a = b$

hence isosceles trapezium

$\Rightarrow$ a cyclic quadrilateral also $\Rightarrow$ **P, S**

(B) $ac = bd \quad \Rightarrow \quad \frac{b}{c} = \frac{a}{d}$



$\left.\begin{array}{l} \tan\theta = \frac{b}{c} \\ \tan\phi = \frac{a}{d} \end{array}\right] \quad \Rightarrow \quad \theta = \phi$

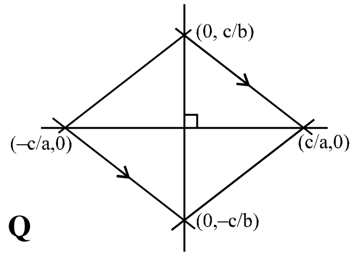
hence cyclic quadrilateral $\Rightarrow$ **P**

(C) $ax \pm by \pm c = 0$

if $y = 0$, $x = \pm \frac{c}{a}$

if $x = 0$, $y = \pm \frac{c}{b}$

$\Rightarrow$ rhombus $\Rightarrow$ **Q**



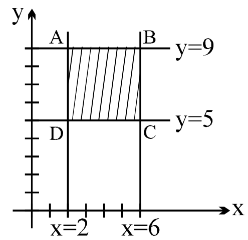
(D) $(x - 6)(x - 2) = 0$

$x = 6$ and $x = 2$

$y^2 - 14y + 45 = 0$

$(y - 9)(y - 5) = 0$

$\Rightarrow$ a square $\Rightarrow$ **P, Q, R**]



[SUBJECTIVE TYPE]

Q.18 If the variable line $3x - 4y + k = 0$ lies between the circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2 + y^2 - 16x - 2y + 61 = 0$ without intersecting or touching either circle, then the range of k is (a, b) where $a, b \in I$. Find the value of $(b - a)$. [Ans. 6] **[6]**

[Sol. The given circle are

$C_1 : (x - 1)^2 + (y - 1)^2 = 1$

and $C_2 : (x - 8)^2 + (y - 1)^2 = 4$

The given line $L : 3x - 4y + k = 0$ will lie between these circles if centres of the circles lie on opposite sides of the line,

i.e. $(3 \cdot 1 - 4 \cdot 1 + k)(3 \cdot 8 - 4 \cdot 1 + k) < 0 \Rightarrow (k - 1)(k + 20) < 0 \Rightarrow k \in (-20, 1)$

Also, the line L will neither touch nor intersect the circle if length of perpendicular drawn from centre to L > corresponding radius

$\therefore$ for $C_1 : \frac{|3 \cdot 1 - 4 \cdot 1 + k|}{5} > 1 \Rightarrow \frac{|k - 1|}{5} > 1$

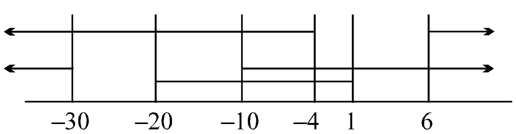
$\Rightarrow$ $k - 1 > 5$ or $k - 1 < -5$

$\Rightarrow$ $k > 6$ or $k < -4$

and for $C_2 : \frac{|3 \cdot 8 - 4 \cdot 1 + k|}{5} > 2 \Rightarrow \frac{|k + 20|}{5} > 2$

$\Rightarrow$ $k + 20 > 10$ or $k + 20 < -10$

$k > -10$ or $k < -30$



$\Rightarrow$ $k \in (-10, -4) \Rightarrow$ $a = -10$ and $b = -4$

$\Rightarrow$ $b - a = -4 + 10 = 6$ **Ans.**]

PRACTICE TEST # 4

M.M. 78 ***Time : 75 Min.***

[STRAIGHT OBJECTIVE TYPE] **[10 × 3 = 30]**

Q.1 If the product of n positive number is unity, then their sum is

(A) a positive (B) divisible by n (C) $n+\frac{1}{n}$ (D*) never less than n

[Sol. Let the number be $x_1, x_2, \ldots\ldots\, x_n$
The A.M. of these numbers ≥ their G.M.

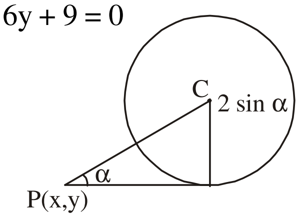
$$\frac{x_1+x_2+\ldots\ldots+x_n}{n} \ge (x_1x_2\ldots\ldots x_n)^{\frac{1}{n}} = 1 \qquad \Rightarrow \qquad x_1+x_2+\ldots\ldots+x_n \ge n$$]

Q.2 If the angle between the tangents drawn from P to the circle $x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$ is 2α, then the locus of P is

(A) $x^2 + y^2 + 4x - 6y + 14 = 0$ (B) $x^2 + y^2 + 4x - 6y - 9 = 0$
(C) $x^2 + y^2 + 4x - 6y - 4 = 0$ (D*) $x^2 + y^2 + 4x - 6y + 9 = 0$

[Sol. $C(-2, 3)$; $R^2 = 4 + 9 - 9\sin^2\alpha - 13\cos^2\alpha = 4\sin^2\alpha$
$R = 2\sin\alpha$

Now use $\sin\alpha = \frac{2\sin\alpha}{CP}$ $\Rightarrow$ Result.]



Q.3 A point P(x, y) moves such that the sum of its distances from the line $2x + y = 1$ and $x + 2y = 1$ is 1. The locus of P is

(A*) a rectangle (B) square (C) parallelogram (D) rhombus

[Sol. $\left|\frac{2h+k-1}{\sqrt{5}}\right| + \left|\frac{h+2k-1}{\sqrt{5}}\right| = 1$

$|2h + k - 1| + |h + 2k - 1| = \sqrt{5}$, now take 4 case an interpret.]

Q.4 Let the H.M. and G.M. of two positive numbers a and b in the ratio 4 : 5 then a : b is

(A) 1 : 2 (B) 2 : 3 (C) 3 : 4 (D*) 1 : 4

[Sol. H.M. = $\frac{2ab}{a+b}$, G.M. = $\sqrt{ab}$

$\frac{\text{H.M.}}{\text{G.M.}} = \frac{2\sqrt{ab}}{a+b} = \frac{4}{5}$ (Given)
$25ab = 4(a+b)^2$ $\Rightarrow$ $4a^2 - 17ab + 4b^2 = 0$
$(4a - b)(a - 4b) = 0$ $\Rightarrow$ $4a = b$ $\Rightarrow$ $a : b = 1 : 4$ **Ans.**]

Q.5 If a, b, c are odd integers, then the equation $ax^2 + bx + c = 0$ cannot have

(A) imaginary roots (B) real root (C) irrational root (D*) rational root

[Sol. Let $x = \frac{m}{n}$, m, n integers $n \ne 0$, be a root

Then $am^2 + bmn + cn^2 = 0$
m, n are odd $\Rightarrow$ odd + odd + odd = 0
m is odd, n is even $\Rightarrow$ odd + even + even = 0

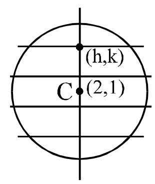
m is even, n is odd $\Rightarrow$ even + even + odd = 0
leading to a contradiction
$\therefore$ there is no rational root.]

Q.6 If two distinct chords, drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$, where $pq \neq 0$, are bisected by the x-axis, then
(A) $p^2 = q^2$ (B) $p^2 = 8q^2$ (C) $p^2 < 9q^2$ (D*) $p^2 > 8q^2$

[Sol. Let $(\alpha,0)$ be the midpoint of the chord. The other end of the chord is $(2\alpha - q\ , -q)$ which lies on the circle.
$\Rightarrow (2\alpha - p, - p)^2 + q^2 = p(2\alpha - p) - q^2$
$\Rightarrow 2\alpha^2 - 3p\alpha, + p^2 + q^2 = 0$
For two values of a, we have
$9p^2 > 8(p^2 + q^2)$ or $p^2 > 8q^2$]

Q.7 Locus of the middle points of a system of parallel chords with slope 2, of the circle $x^2 + y^2 - 4x - 2y - 4 = 0$, has the equation
(A*) $x + 2y - 4 = 0$ (B) $x - 2y = 0$ (C) $2x - y - 3 = 0$ (D) $2x + y - 5 = 0$

[Hint: Locus will be a line with slope – 1/2
and passing through the centre (2, 1) of the circle

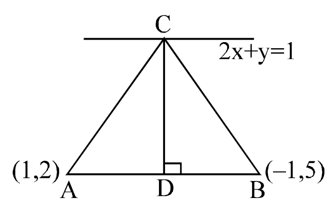


$$y - 1 = -\frac{1}{2}(x - 2)$$

$2y - 2 = - x + 2 \Rightarrow x + 2y - 4 = 0$ Ans.]

Q.8 A(1, 2), B(–1, 5) are two vertices of a triangle whose are is 5 units. If the third vertex C lies on the line $2x + y = 1$, then C is
(A) (0, 1) or (1, 21) (B*) (5, – 9) or (–15, 31)
(C) (2, – 3) or (3, –5) (D) (7, – 13) or (– 7, 15)

[Sol. A(1, 2) ; B (–1, 5)
C point $(\alpha, 1 - 2\alpha)$



$|AB| = \sqrt{13}$

$$(y - 2) = \frac{3}{-2}(x - 1) \Rightarrow 3x + 2y - 7 = 0$$

$$|CD| = \frac{|3\alpha + 2(1 - 2\alpha) - 7|}{\sqrt{13}} = \frac{|-\alpha - 5|}{\sqrt{13}}$$

$$\left|\frac{1}{2}|CD| \times |AB|\right| = 5 \Rightarrow \frac{1}{2}\frac{|\alpha + 5|}{\sqrt{13}} \times \sqrt{13} = 5 \Rightarrow |\alpha + 5| = 10 \Rightarrow \alpha = 5 \text{ or } -15$$

$C \to (5, - 9)$ or $(- 15, 31)$ **Ans.**]

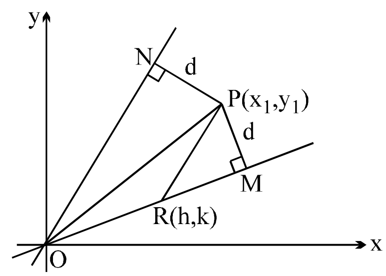
Q.9 The distance of the point (x_1, y_1) from each of the two straight lines through the origin is d. The equation of the two straight lines is
(A*) $(xy_1 - yx_1)^2 = d^2(x^2 + y^2)$ (B) $d^2(xy_1 - yx_1)^2 = x^2 + y^2$
(C) $d^2(xy_1 + yx_1)^2 = x^2 + y^2$ (D) $(xy_1 + yx_1)^2 = d^2(x^2 + y^2)$

[Sol. Let R (h, k) be any point on OM

$$\text{Area of } \Delta \text{ OPR} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ h & k & 1 \\ 0 & 0 & 1 \end{vmatrix} = \frac{1}{2}\left|(kx_1 - hy_1)\right|$$

also $\quad \ldots\ldots \Delta \text{OPR} = \dfrac{\sqrt{h^2+k^2}\cdot d}{2}$

$\therefore \quad \dfrac{1}{2}\left|(kx_1 - hy_1)\right| = \dfrac{\sqrt{h^2+k^2}\cdot d}{2}$



locus of (h, k) is

$(xy_1 - yx_1)^2 = d^2(x^2 + y^2)$ **Ans.**

Alternatively: Let the line through (0, 0) be $y = mx$

$\therefore \quad d = \left|\dfrac{mx_1 - y_1}{\sqrt{1+m^2}}\right| = m^2(x_1^2 - d^2) - 2mx_1y_1 + y_1^2 - d^2 = 0$

replacing m by y/x

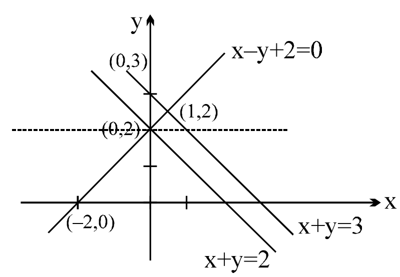
$x^2(y_1^2 - d^2) - 2\,xy\,x_1y_1 + y^2(x_1^2 - d^2) = 0$

$(xy_1 - yx_1)^2 = d^2(x^2 + y^2)$ **Ans.**]

Q.10 Area of the triangle formed by the line $x + y = 3$ and the angle bisectors of the line pair $x^2 - y^2 + 4y - 4 = 0$ is

(A*) 1/2 (B) 1

(C) 3/2 (D) 2



[Sol. $x^2 - (y^2 - 4y + 4) = 0$

$\Rightarrow \quad x^2 - (y-2)^2 = 0$

$\Rightarrow \quad (x + y - 2)(x - y + 2) = 0$

Area $= \dfrac{1\cdot 1}{2} = \dfrac{1}{2}$ **Ans.**]

[COMPREHENSION TYPE] [3 × 3 = 9]

Paragraph for Question Nos. 11 to 13

Consider a general equation of degree 2, as

$\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$

Q.11 The value of 'λ' for which the line pair represents a pair of straight lines is

(A) 1 (B*) 2 (C) 3/2 (D) 3

Q.12 For the value of λ obtained in above question, if $L_1 = 0$ and $L_2 = 0$ are the lines denoted by the given line pair then the product of the abscissa and ordinate of their point of intersection is

(A) 18 (B) 28 (C*) 35 (D) 25

Q.13 If θ is the acute angle between $L_1 = 0$ and $L_2 = 0$ then θ lies in the interval

(A) (45°, 60°) (B) (30°, 45°) (C) (15°, 30°) (D*) (0, 15°)

[Sol.

(i) $a = \lambda;\ h = -5;\ b = 12;\ g = \dfrac{5}{2};\ f = -8,\ c = -3$

$\lambda(12)(-3) + 2(-8)\left(\dfrac{5}{2}\right)(-5) - \lambda(64) - \left(\dfrac{25}{4}\right)\cdot 12 + 3\cdot 25 = 0$

$-36\lambda + 200 - 64\lambda - 75 + 75 = 0 \quad \Rightarrow \quad 100\lambda = 200 \quad \Rightarrow \quad \lambda = 2$ Ans.

(ii) $2x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$

consider the homogeneous part

$2x^2 - 10xy + 12y^2$

$2x^2 - 6xy - 4xy + 12y^2$ or $2x(x - 3y) - 4y(x - 3y)$ or $(x - 3y)(x - 2y)$

$\therefore$ $2x^2 - 10xy + 12y^2 + 5x - 16y - 3 \equiv (2x - 6y + A)(x - 2y + B)$

solving $A = -1$; $B = 3$

hence lines are $2x - 6y - 1 = 0$ and $x - 2y + 3 = 0$

solving intersection point $\left(-10, -\frac{7}{2}\right)$

$\therefore$ product = 35 **Ans.**

(iii) $\tan\theta = \dfrac{2\sqrt{h^2 - ab}}{a + b} = \dfrac{2\sqrt{25 - 24}}{14} = \dfrac{1}{7}$ $\Rightarrow$ $\theta \in (0, 15°)$ **Ans.**]

[REASONING TYPE] [1 × 3 = 3]

Q.14 A circle is circumscribed about an equilateral triangle ABC and a point P on the minor arc joining A and B, is chosen. Let x = PA, y = PB and z = PC. (z is larger than both x and y.)

Statement-1: Each of the possibilities (x + y) greater than z, equal to z or less than z is possible for some P.

because

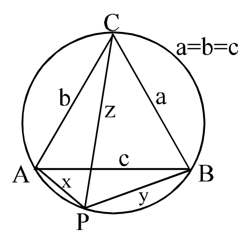
Statement-2: In a triangle ABC, sum of the two sides of a triangle is greater than the third and the third side is greater than the difference of the two.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D*) Statement-1 is false, statement-2 is true.



[Sol. Using Tolemy's theorem for a cyclic quadrilateral

(z) (AB) = ax + by

$z \cdot c = ax + by$

but $a = b = c$

hence $x + y = z$ is true always

$\Rightarrow$ S-1 is false and S-2 is true]

[MATCH THE COLUMN] [(3+3+3+3)×2=24]

Q.15 Set of family of lines are described in column-I and their mathematical equation are given in column-II. Match the entry of column-I with suitable entry of column-II. (*m* and *a* are parameters)

	Column-I		**Column-II**
(A)	having gradient 3	(P)	$mx - y + 3 - 2m = 0$
(B)	having y intercept three times the x-intercept	(Q)	$mx - y + 3m = 0$
(C)	having x intercept (–3)	(R)	$3x + y = 3a$
(D)	concurrent at (2, 3)	(S)	$3x - y + a = 0$

[**Ans.** (A) S; (B) R; (C) Q; (D) P]

[Sol. can be easily analysed.]

Q.16 **Column-I** **Column-II**

(A) Let 'P' be a point inside the triangle ABC and is equidistant from its sides. DEF is a triangle obtained by the intersection of the external angle bisectors of the angles of the ΔABC. With respect to the triangle DEF point P is its — (P) centroid

(B) Let 'Q' be a point inside the triangle ABC — (Q) orthocentre

If $(AQ)\sin\frac{A}{2} = (BQ)\sin\frac{B}{2} = (CQ)\sin\frac{C}{2}$ then with respect to the triangle ABC, Q is its

(C) Let 'S' be a point in the plane of the triangle ABC. If the point is such that infinite normals can be drawn from it on the circle passing through A, B and C then with respect to the triangle ABC, S is its — (R) incentre

(D) Let ABC be a triangle. D is some point on the side BC such that the line segments parallel to BC with their extremities on AB and AC get bisected by AD. Point E and F are similarly obtained on CA and AB. If segments AD, BE and CF are concurrent at a point R then with respect to the triangle ABC, R is its — (S) circumcentre

[Ans. (A) Q; (B) R; (C) S; (D) P]

[SUBJECTIVE TYPE]

Q.17 If a, b, c are positive, then find the minimum value of $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$. **[6]**

[Ans. 9]

[Sol. For a, b, c, A.M. = $\frac{a+b+c}{3}$, H.M. = $\frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$

A.M. ≥ G.M. ≥ H.M. ⇒ $\frac{a+b+c}{3} \ge \frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$ ⇒ $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \ge 9$]

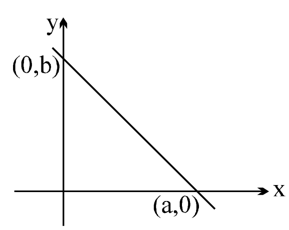
Q.18 Find the number of straight lines parallel to the line 3x + 6y + 7 = 0 and have intercept of length 10 between the coordinate axes. [Ans. 2] **[6]**

[Sol. Slope of the given line is – 1/3

let one line is $\frac{x}{a}+\frac{y}{b} = 1$

∴ slope = $-\frac{b}{a}$

⇒ $-\frac{b}{a} = -\frac{1}{3}$ ⇒ 3b = a ....(1)



also given $a^2 + b^2 = 100$ ....(2)

(1) and (2) ⇒ b = ± $\sqrt{10}$

b = $\sqrt{10}$; a = $3\sqrt{10}$

b = – $\sqrt{10}$; a = – $3\sqrt{10}$

∴ Note a and b must be of same sign]

PRACTICE TEST # 5

M.M. 79 ***Time : 70 Min.***

[STRAIGHT OBJECTIVE TYPE] **[9 × 3 = 27]**

Q.1 A square is inscribed in the cirle $x^2 + y^2 – 2x + 4y + 33 = 0$. Its sides are parallel to the coordinate axes. Then one vertex of the square is

(A) $\left(1+\sqrt{2}, –2\right)$ (B) $\left(1-\sqrt{2}, –2\right)$ (C) $\left(1, –2+\sqrt{2}\right)$ (D*) None

[Sol. The centre of the circle is (1, –2) and radius $\sqrt{2}$. The diagonal of the square is $2\sqrt{2}$ and side is 2. The vertices are (0,–3), (2, –3) (2,–1), (0,–1).]

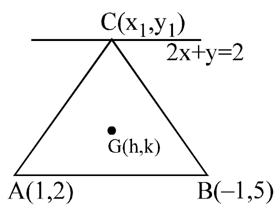
Q.2 If $4^3 = 8^{1+|\cos x|+\cos^2 x+........\infty}$, then the number of values of x in [0, 2π], is

(A) 1 (B) 2 (C) 3 (D*) 4

Q.3 A(1, 2), B(–1, 5) are two vertices of a triangle ABC whose third vertex C lies on the line 2x + y = 2. The locus of the centroid of the triangle is

(A*) 2x + y = 3 (B) x + 2y = 3 (C) 2x – y = 3 (D) – 2x – y = 3

[Sol. $h = \dfrac{x_1 + 1 + (-1)}{3}$; $k = \dfrac{5 + 2 + y_1}{3}$



$x_1 = 3h$; $y_1 = 3k – 7$

This lies on line 2x + y = 2

2(3x) + 3k – 7 = 2

⇒ 6x + 3y = 9

⇒ 2x + y = 3 **Ans.**]

Q.4 If a, b, c, d and p are distinct real numbers such that $(a^2 + b^2 + c^2)p^2 – 2(ab + bc + cd)p + b^2 + c^2 + d^2 \le 0$. Then a, b, c, d are

(A) in A.P. (B*) in G.P. (C) in H.P. (D) satisfy ab = cd

[Sol. $(a^2 + b^2 + c^2)p^2 – 2(ab + bc + cd)p + b^2 + c^2 + d^2 \le 0$

⇒ $(ap – b)^2 + (bp – c)^2 + (cp – d)^2 \le 0$

The sum of squares cannot be negative

∴ $(ap – b)^2 + (bp – c)^2 + (cp – d)^2 = 0$

ap – b = bp – c = cp – d = 0

$p = \dfrac{b}{a} = \dfrac{c}{b} = \dfrac{d}{c}$ ⇒ a, b, c, d are in G.P.]

Q.5 A root of the equation $(a + b)(ax + b)(a – bx) = (a^2x – b)(a + bx)$ is

(A) $\dfrac{a+2b}{2a+b}$ (B) $\dfrac{2a+b}{a+2b}$ (C) $\dfrac{a-2b}{2a-b}$ (D*) $-\left(\dfrac{a+2b}{2a+b}\right)$

[Sol. Simplifying, the equation becomes

$(2a + b)x^2 – (a – b)x – (a + 2b) = 0$

The sum of the coefficients = 0 ⇒ x = 1 is a root.

The other root = $-\left(\dfrac{a+2b}{2a+b}\right)$]

Q.6 A rhombus is inscribed in the region common to the two circles $x^2 + y^2 - 4x - 12 = 0$ and $x^2 + y^2 + 4x - 12 = 0$ with two of its vertices on the line joining the centres of the circles. The area of the rhombous is :

(A*) $8\sqrt{3}$ sq.units (B) $4\sqrt{3}$ sq.units

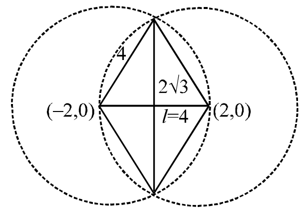
(C) $16\sqrt{3}$ sq.units (D) none

[Hint : circles with centre (2, 0) and (−2, 0) each with radius 4

$\Rightarrow$ y−axis is their common chord.

The inscribed rhombus has its diagonals equal to 4 and $4\sqrt{3}$

$\therefore \quad A = \frac{d_1 d_2}{2} = 8\sqrt{3}$]



Q.7 The locus of the centre of circle which touches externally the circle $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches the y -axis is

(A) $x^2 - 6x - 10y + 14 = 0$ (B) $x^2 - 10x - 6y + 14 = 0$
(C) $y^2 - 6x - 10y + 14 = 0$ (D*) $y^2 - 10x - 6y + 14 = 0$

[Sol. If (x_1, y_1) is the centre of the circle, then
$(x - x_1)^2 + (y - y_1)^2 = x_1^2$
It touches the circle with centre (3,30 and radius 2. The desired locus is
$\therefore (x - 3)^2 + (y - 3)^2 = (x + 2)^2$
or $y^2 - 10x - 6x + 14 = 0$]

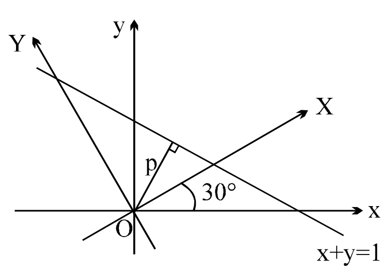
Q.8 The coordinates axes are rotated about the origin 'O' in the counter clockwise direction through an angle of $\pi/6$. If *a* and *b* are intercepts made on the new axes by a straight line whose equation referred to the old axes is x + y = 1 then the value of $\frac{1}{a^2} + \frac{1}{b^2}$ is equal to

(A) 1 (B*) 2 (C) 4 (D) $\frac{1}{2}$

[Sol. Equation of line w.r.t. new axes

$$\frac{X}{a} + \frac{Y}{b} = 1$$

$$p = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$$

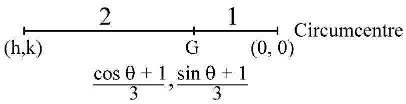


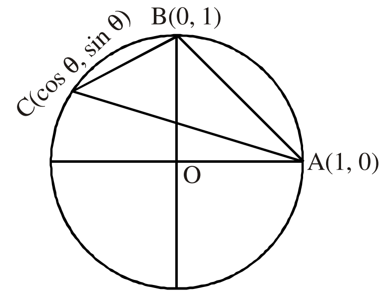
$\Rightarrow \quad \frac{1}{a^2} + \frac{1}{b^2} = 2$ **Ans.**]

Q.9 A(1, 0) and B(0, 1) and two fixed points on the circle $x^2 + y^2 = 1$. C is a variable point on this circle. As C moves, the locus of the orthocentre of the triangle ABC is

(A*) $x^2 + y^2 - 2x - 2y + 1 = 0$
(B) $x^2 + y^2 - x - y = 0$
(C) $x^2 + y^2 = 4$
(D) $x^2 + y^2 + 2x - 2y + 1 = 0$

[Sol. Let C (cos θ, sin θ); H(h, k) is the orthocentre of the Δ ABC





$h = 1 + \cos\theta$

$k = 1 + \sin\theta$

$(x - 1)^2 + (y - 1)^2 = 1$

$x^2 + y^2 - 2x - 2y + 1 = 0$]

[COMPREHENSION TYPE] [3 × 3 = 9]

Paragraph for question nos. 25 to 27

Consider 3 circles

$S_1 : x^2 + y^2 + 2x - 3 = 0$

$S_2 : x^2 + y^2 - 1 = 0$

$S_3 : x^2 + y^2 + 2y - 3 = 0$

Q.10 The radius of the circle which bisect the circumferences of the circles $S_1 = 0$; $S_2 = 0$; $S_3 = 0$ is

(A) 2 (B) $2\sqrt{2}$ (C*) 3 (D) $\sqrt{10}$

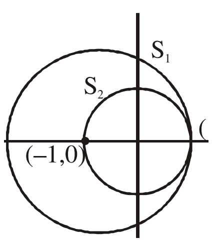
Q.11 If the circle S = 0 is orthogonal to $S_1 = 0$; $S_2 = 0$ and $S_3 = 0$ and has its centre at (a, b) and radius equals to 'r' then the value of $(a + b + r)$ equals

(A) 0 (B) 1 (C) 2 (D*) 3

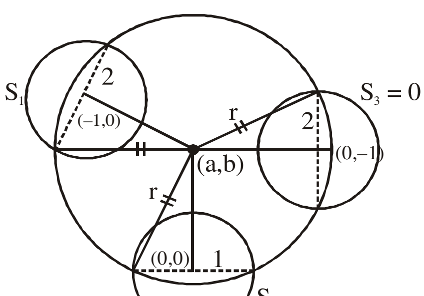
Q.12 The radius of the circle touching $S_1 = 0$ and $S_2 = 0$ at (1, 0) and passing through (3, 2) is

(A) 1 (B) $\sqrt{12}$ (C*) 2 (D) $2\sqrt{2}$

[Sol.



(i)



$r^2 = a^2 + b^2 + 1 = (a + 1)^2 + b^2 + 4$ and $(a + 1)^2 + b^2 + 4 = a^2 + (b + 1)^2 + 4$

∴ $2a + 4 = 0$ $2a = 2b$

$a = -2$ $b = -2$

$r^2 = 9 \Rightarrow r = 3$ **Ans.**

(ii) $S_1 - S_2 = 0 \Rightarrow x = 1$

$S_2 - S_3 \Rightarrow y = 1$

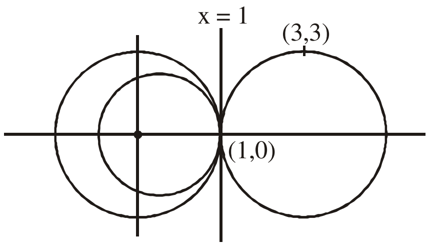
$\therefore$ Radical centre = (1, 1)

radius $L_T = \sqrt{S_1} = 1$

$\therefore$ equation of circle is $(x-1)^2 + (y-1)^2 = 1$

$\Rightarrow$ radius = 1 and a = 1 ; b = 1 $\Rightarrow$ a + b + r = 3 **Ans.**

(iii)



family of circles touches the line x – 1 = 0 at (1, 0) is

$(x-1)^2 + (y-0)^2 + \lambda(x-1) = 0$

passing through (3, 2) $\Rightarrow$ $4 + 4 + 2\lambda = 0 \Rightarrow$ $\lambda = -4$

$\therefore$ $x^2 + y^2 - 6x + 5 = 0$

$\therefore$ radius $\sqrt{9-5} = 2$ **Ans.**]

[REASONING TYPE] **[1 × 3 = 3]**

Q.13 Consider the circle $C : x^2 + y^2 - 2x - 2y - 23 = 0$ and a point P(3, 4).

Statement-1: No normal can be drawn to the circle C, passing through (3, 4).

because

Statement-2: Point P lies inside the given circle, C.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D*) Statement-1 is false, statement-2 is true.

[MULTIPLE OBJECTIVE TYPE] **[1 × 4 = 4]**

Q.14 Let L_1 be a line passing through the origin and L_2 be the line x + y = 1. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$

(A) x + y = 0 (B*) x – y = 0 (C*) x + 7y = 0 (D) x – 7y = 0

[Sol. The chords are of equal length, then the distances of the centre from the lines are equal.

Let L_1 be y – mx = 0. Centre is $\left(\frac{1}{2}, -\frac{3}{2}\right)$

$$\frac{\left|-\frac{3}{2} - \frac{m}{2}\right|}{\sqrt{m^2+1}} = \frac{\left|\frac{1}{2} - \frac{3}{2} - 1\right|}{\sqrt{2}} \Rightarrow 7m^2 - 6x - 1 = 0$$

$\Rightarrow m = 1, -\frac{1}{7}$]

[MATCH THE COLUMN] [(3+3+3+3)×2=24]

Q.15

Column-I		Column-II	
(A)	The sum $\sum_{r=1}^{100} r^2 \tan\left(\frac{2r-1)\pi}{4}\right)$ is equal to	(P)	– 5151
(B)	Solution of the equation $\cos^4 x = \cos 2x$ which lie in the interval [0, 314] is kπ where k equals	(Q)	– 5050
(C)	Sum of the integral solutions of the inequality $\log_{1/\sqrt{5}}(6^{x+1} - 36^x) \ge -2$ which lie in the interval [–101, 0]	(R)	5049
		(S)	4950
(D)	Let $P(n) = \log_2 3 \cdot \log_3 4 \cdot \log_4 5 \ldots\ldots \log_{n-1}(n)$ then the value of $\sum_{k=2}^{100} P(2^k)$ equals		

[Ans. (A) Q; (B) S; (C) P ; (D) R]

[Sol. (A) $S = 1^2 - 2^2 + 3^2 - 4^2 + \ldots + 99^2 - 100^2$

$= -[(2^2 - 1^2) + (4^2 - 3^2) + \ldots + (100^2 - 99^2)]$

$= -[1 + 2 + 3 + 4 + \ldots + 99 + 100] = -5050 \Rightarrow$ **(Q) Ans.**

(B) $\cos^4 x = 2\cos^2 x - 1$

$1 + \cos^4 x - 2\cos^2 x = 0$

$(1 - \cos^2 x)^2 = 0$

$\sin^2 x = 0$

$x = \pi[1 + 2 + 3 + \ldots + 99]$

$= 4950\pi \Rightarrow k = 4950 \Rightarrow$ **(S) Ans.**

(C) $0 < (6^{x+1} - 36^x) \le \left(\frac{1}{\sqrt{5}}\right)^{-2}$

$6 \cdot 6^x - 6^{2x} \le 5$

$6^{2x} - 6 \cdot 6^x + 5 \ge 0$

$(6^x - 1)(6^x - 5) \ge 0$

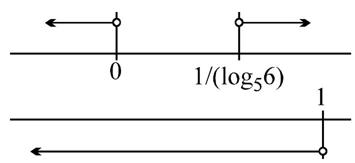
$6^x \ge 5$ or $6^x \le 1 \Rightarrow x \ge \frac{1}{\log_5 6}$ or $x \le 0$ ....(1)

$6^{x+1} - 36^x > 0$

$6 - 6^x > 0 \Rightarrow 6 > 6^x$

$\therefore$ $x < 1$ ....(2)

From (1) and (2), we have



$x \ge \frac{1}{\log_5 6}$ or $x \le 0$

$x \in (-\infty, 0] \cup [\log_6 5, 1) \Rightarrow$ **(P) Ans.**

(D) $P(n) = \log_2 n$

$P(2^k) = \log_2 2^k = k$

$\therefore \sum_{k=2}^{100} (k) = 5049 \Rightarrow$ **(R) Ans.**]

Q.16 **Column-I** / **Column-II**

	Column-I		Column-II
(A)	Two intersecting circles	(P)	have a common tangent
(B)	Two circles touching each other	(Q)	have a common normal
(C)	Two non concentric circles, one strictly inside the other	(R)	do not have a common normal
(D)	Two concentric circles of different radii	(S)	do not have a radical axis.

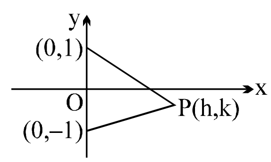
[Ans. (A) P, Q; (B) P, Q; (C) Q; (D) Q, S]

[SUBJECTIVE]

Q.17 A(0, 1) and B(0, – 1) are 2 points if a variable point P moves such that sum of its distance from A and B is 4. Then the locus of P is the equation of the form of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Find the value of $(a^2 + b^2)$ is .

[Ans. 7] **[6]**

[Sol. $\sqrt{h^2+(k-1)^2}+\sqrt{h^2+(k+1)^2}=4$



$h^2 + (k – 1)^2 = 16 + h^2 + (k + 1)^2 – 8\sqrt{h^2+(k+1)^2}$

$16 + 4k = 8\sqrt{h^2+(k+1)^2} \quad\Rightarrow\quad 4 + k = 2\sqrt{h^2+(k+1)^2}$

$16 + k^2 + 8k = 4h^2 + 4(k + 1)^2$

$4h^2 + 3k^2 = 12$

$\frac{h^2}{3}+\frac{k^2}{4}=1 \quad\Rightarrow\quad \frac{x^2}{3}+\frac{y^2}{4}=1 \quad\Rightarrow\quad a^2 = 3 \text{ and } b^2 = 4 \quad\Rightarrow\quad 3 + 4 = 7$ Ans]

Q.18 Find the product of all the values of x satisfying the equation $\left(5+2\sqrt{6}\right)^{x^2-3}+\left(5-2\sqrt{6}\right)^{x^2-3}=10.$

[6]

[Ans. 8]

[Sol. Since $5-2\sqrt{6}=\frac{1}{5+2\sqrt{6}}$, we have $t+\frac{1}{t}=10$ where $t=\left(5+2\sqrt{6}\right)^{x^2-3}$ ....(1)

$\Rightarrow \quad t^2 – 10t + 1 = 0 \quad\Rightarrow\quad t = 5\pm 2\sqrt{6}$

or $\quad t=\left(5+2\sqrt{6}\right)^{\pm 1}$ ....(2)

(1), (2) $\Rightarrow \quad x^2 – 3 = \pm 1 \quad\Rightarrow\quad x^2 = 2, 4$

$\Rightarrow \quad x=-\sqrt{2},\sqrt{2},-2,2;\quad\therefore$ product = 8 **Ans.**]

PRACTICE TEST # 6

M.M. 77 ***Time : 90 Min.***

[STRAIGHT OBJECTIVE TYPE] **[12 × 3 = 36]**

Q.1 The sum of the infinite series $1+\frac{2}{3}+\frac{3}{3^2}+\frac{4}{3^3}+.......$ is

(A) $\frac{7}{4}$ (B) 2 (C) $\frac{8}{3}$ (D*) $\frac{9}{4}$

Q.2 For real values of x, the function $\frac{\sin x \cos 3x}{\sin 3x \cos x}$ does not take values

(A) between – 1 and 1 (B) between 0 and 2

(C*) between $\frac{1}{3}$ and 3 (D) between 0 and $\frac{1}{3}$

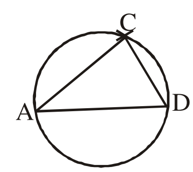
[Sol. $y = \frac{\tan x}{\tan 3x} = \frac{1-3t^2}{3-t^2}$, $t = \tan x$ as $\tan x \neq 0$, $y \neq 1/3$

$y(3 - t^2) = 1 - 3t^2$

$\Rightarrow \quad 0 \le t^2 = \frac{3y-1}{y-3} \quad \Rightarrow \quad (3y-1)(y-3) \ge 0 \quad (y \neq 3)$

$\therefore \quad y \in \left(-\infty, \frac{1}{3}\right) \cup (3, \infty)$]

Q.3 AB is a diameter of a circle and C is any point on the circumference of the circle. Then



(A*) Area of ΔABC is maximum when it is isosceles.
(B) Area of ΔABC is minimum when it is isosceles.
(C) Perimeter of ΔABC is minimum when it is isosceles.
(D) None

[Sol. Area of ΔABC is maximum when C is farthest from AB, i.e. when it is isosceles.]

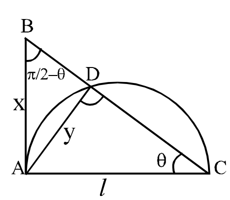
Q.4 The sides of a right angled triangle are in G.P. The ratio of the longest side to the shortest side is

(A) $\frac{\sqrt{3}+1}{2}$ (B) $\sqrt{3}$ (C) $\frac{\sqrt{5}-1}{2}$ (D*) $\frac{\sqrt{5}+1}{2}$

Q.5 In a right triangle ABC, right angled at A, on the leg AC as diameter, a semicircle is described. The chord joining A with the point of intersection D of the hypotenuse and the semicircle, then the length AC equals to

(A) $\frac{AB \cdot AD}{\sqrt{AB^2 + AD^2}}$ (B) $\frac{AB \cdot AD}{AB + AD}$ (C) $\sqrt{AB \cdot AD}$ (D*) $\frac{AB \cdot AD}{\sqrt{AB^2 - AD^2}}$

[Sol. $l \cdot x = y\sqrt{l^2 + x^2}$ where $l = AC$; $x = AB$, $y = AD$



$l^2 x^2 = y^2(l^2 + x^2)$

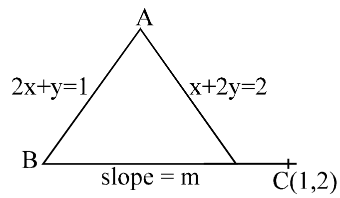
$l^2(x^2 - y^2) = x^2y^2$

$l = \frac{xy}{\sqrt{x^2 - y^2}} = \frac{AB \cdot AD}{\sqrt{AB^2 - AD^2}}$ Ans.]

Q.6 ABC is an isoscele triangle with AB = AC. The equation of the sides AB and AC are 2x + y = 1 and x + 2y = 2. The sides BC passes through the point (1, 2) and makes positive intercept on the x-axis. The equation of BC is

(A) x – y + 1 = 0 (B*) x + y – 3 = 0 (C) 2x + y – 4 = 0 (D) x – 2y + 3 = 0

[Sol. Slope of AB = – 2 ; slope of AC = $\frac{-1}{2}$; slope of BC = m



$$\frac{m+2}{1-2m} = \frac{-\frac{1}{2}-m}{1-\frac{1}{2}m} \quad \Rightarrow \quad 4 - m^2 = -(1 - 4m^2) = 4m^2 - 1$$

$5m^2 = 5 \quad \Rightarrow \quad m = \pm 1$

$(y - 2) = 1(x - 1)$ or $(y - 2) = -1(x - 1)$

x-intercept x = – 1 x = 3 **Ans.**]

Q.7 The number of tangents that can be drawn from the point $\left(\frac{5}{2}, 1\right)$ to the circle passing through the points $(1, \sqrt{3})$, $(1, -\sqrt{3})$ and $(3, -\sqrt{3})$ is

(A) 1 (B*) 0 (C) 2 (D) None

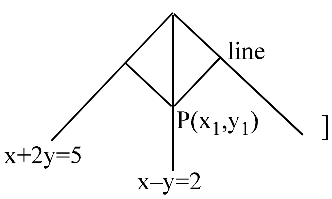
[Sol. The triangle is right angled. Its circum circle is $x^2 + y^2 - 4x = 0$ $\left(\frac{5}{2}\right)^2 + 1 - 4 \cdot \frac{5}{2} < 0$ The point is inside the circle.]

Q.8 The image of the line x + 2y = 5 in the line x – y = 2, is

(A*) 2x + y = 7 (B) x + 2y = 5 (C) 2x + 3y = 9 (D) 2x – 3y = 3

[Sol. Image is $x + 2y - 5 + \lambda(x - y - 2) = 0$

now equate perpendicular distance



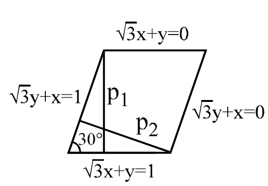
]

Q.9 The area of the quadrilateral formed by the lines $\sqrt{3}x + y = 0$, $\sqrt{3}y + x = 0$, $\sqrt{3}x + y = 1$, $\sqrt{3}y + x = 1$ is

(A) 1 (B*) $\frac{1}{2}$ (C) $\sqrt{2}$ (D) 2

[Sol. $p_1 = \frac{1}{2}$; $p_2 = \frac{1}{2}$

Hence it is a rhombus

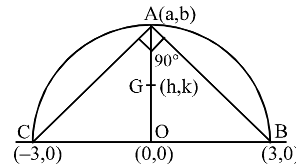


Area is $\frac{p_1 p_2}{\sin\theta}$

$(\theta = 30°) = \frac{1}{4} \cdot \frac{2}{1} = \frac{1}{2}$ **Ans.**]

Q.10 B and C are fixed points having co–ordinates (3, 0) and (–3, 0) respectively. If the vertical angle BAC is 90°, then the locus of the centroid of the ΔABC has the equation :
(A*) $x^2 + y^2 = 1$ (B) $x^2 + y^2 = 2$ (C) $9(x^2 + y^2) = 1$ (D) $9(x^2 + y^2) = 4$

[Hint : Let A (a, b) and G (h. k) Now A, G, O are collinear



$\Rightarrow h = \dfrac{2.0 + a}{3} \Rightarrow a = 3h$ and similarly $b = 3k$.

Now (a, b) lies on the circle $x^2 + y^2 = 9 \Rightarrow$ A]

Q.11 Let a, b, c three numbers between 2 and 18 such that their sum is 25. If 2, a, b are in A.P. and b, c, 18 are in G.P., then 'c' equal
(A) 10 (B*) 12 (C) 14 (D) 16

[Sol. $a + b + c = 25$ ....(1)
2, a, b are in A.P. $\Rightarrow$ $2 + b = 2a$ ....(2)
b, c, 18 are in G.P. $\Rightarrow$ $c^2 = 18b$ ....(3)
Eliminating a and b from (1) to (3)

$$a = 1 + \frac{b}{2} = 1 + \frac{c^2}{36}, \quad b = \frac{c^2}{18}$$

$$1 + \frac{c^2}{36} + \frac{c^2}{18} + c = 25 \quad \Rightarrow \quad c^2 + 12c - 288 = 0 \quad \Rightarrow \quad c = 12, -24$$

But 'c' lies between 2 and 18
$\therefore$ c = 12 **Ans.**]

Q.12 If the roots of $x^2 + px + q = 0$ are tan 30° and tan 15°, then $(2 + q - p)$ equals
(A) 0 (B) 1 (C) 2 (D*) 3

[Sol. $-p = \tan 30° + \tan 15° = \dfrac{1}{\sqrt{3}} + 2 - \sqrt{3} = \dfrac{2\sqrt{3} - 2}{\sqrt{3}}$

$q = \tan 30° \tan 15° = \dfrac{1}{\sqrt{3}}\left(2 - \sqrt{3}\right) = \dfrac{2 - \sqrt{3}}{\sqrt{3}}$

$2 + q - p = 2 + \dfrac{2 - \sqrt{3} + 2\sqrt{3} - 2}{\sqrt{3}} = 3$ **Ans.**]

[REASONING TYPE] [1 × 3 = 3]

Q.13 Consider the lines
L : $(k + 7)x - (k - 1)y - 4(k - 5) = 0$ where k is a parameter
and the circle
C : $x^2 + y^2 + 4x + 12y - 60 = 0$
Statement-1: Every member of L intersects the circle 'C' at an angle of 90°
because
Statement-2: Every member of L is tangent to the circle C.
(A) Statement-1 is true, statement-2 is true; statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true; statement-2 is NOT the correct explanation for statement-1.
(C*) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

[Exp. Centre (–2, –6). Substituting in L
$-2(k + 7) + 6(k - 1) - 4(k - 5) = (-2k + 6k - 4k) - 14 - 6 + 20 = 0$
Hence every member of L passing through the centre of the circle $\Rightarrow$ cuts it at 90°.
Hence S-1 is true and S-2 is false.]

[MULTIPLE OBJECTIVE TYPE] **[2 × 4 = 8]**

Q.14 Consider the points O (0, 0), A (0, 1) and B (1, 1) in the x-y plane. Suppose that points C (x, 1) and D (1, y) are chosen such that $0 < x < 1$ and such that O, C and D are collinear. Let sum of the area of triangles OAC and BCD be denoted by 'S' then which of the following is/are correct?

(A*) Minimum value of S is irrational lying in (1/3, 1/2)

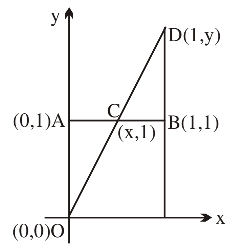
(B) Minimum value of S is irrational in (2/3, 1)

(C*) The value of x for minimum value of S lies in (2/3, 1)

(D) The value of x for minimum values of S lies in (1/3, 1/2)

[Sol. S = Area of Δ OAC + area of Δ BCD

$$= \frac{1\cdot x}{2} + \frac{(1-x)(y-1)}{2} \qquad 0 < x < 1$$



$$S = \frac{x}{2} - \frac{(x-1)(y-1)}{2} \quad ....(1)$$

Now Δ's CBD and OCA are similar

$$\therefore \quad \frac{y-1}{1} = \frac{1-x}{x}$$

$$y = 1 + \frac{1-x}{x} = \frac{1}{x}$$

$$S = \frac{x}{2} - \frac{(x-1)((1/x)-1)}{2} = \frac{x}{2} + \frac{(x-1)^2}{2x} = \frac{x^2 + (x-1)^2}{2x} = \frac{2x^2 - 2x + 1}{2x}$$

$$= x + \frac{1}{2x} - 1 = \left(\sqrt{x} - \frac{1}{\sqrt{2x}}\right)^2 - 1 + \sqrt{2}$$

$\therefore$ A is minimum if $\sqrt{x} = \frac{1}{\sqrt{2x}}$ i.e. $x = \frac{1}{\sqrt{2}}$ which lies in (2/3, 1)

and $A_{min} = \sqrt{2} - 1$ which lies in (1/3, 1/2) $\Rightarrow$ (A) & (C)]

Q.15 If $5x - y$, $2x + y$, $x + 2y$ are in A.P. and $(x-1)^2$, $(xy+1)$, $(y+1)^2$ are in G.P., $x \neq 0$, then $(x + y)$ equals

(A*) $\frac{3}{4}$ (B) 3 (C) – 5 (D*) – 6

[Sol. $5x - y + x + 2y = 2(2x + y) \Rightarrow 2x = y$]

$(x-1)^2(y+1)^2 = (xy+1)^2 \Rightarrow (x-1)(2x+1) = \pm (xy+1)$

$-x - 1 = 1 \Rightarrow x = -2, y = -4$

Also $2x^2 - x - 1 = -2x^2 - 1 \Rightarrow x = \frac{1}{4}, \; y = \frac{1}{2}$

$\therefore \quad x + y = -6$ or $\frac{3}{4}$]

[MATCH THE COLUMN]

[(3+3+3+3)×2=24]

Q.16 **Column-I** — **Column-II**

(A) The four lines $3x - 4y + 11 = 0$; $3x - 4y - 9 = 0$; $4x + 3y + 3 = 0$ and $4x + 3y - 17 = 0$ enclose a figure which is

(B) The lines $2x + y = 1$, $x + 2y = 1$, $2x + y = 3$ and $x + 2y = 3$ form a figure which is

(C) If 'O' is the origin, P is the intersection of the lines $2x^2 - 7xy + 3y^2 + 5x + 10y - 25 = 0$, A and B are the points in which these lines are cut by the line $x + 2y - 5 = 0$, then the points O, A, P, B (in some order) are the vertices of

(P) a quadrilateral which is neither a parallelogram nor a trapezium nor a kite.

(Q) a parallelogram which is neither a rectangle nor a rhombus

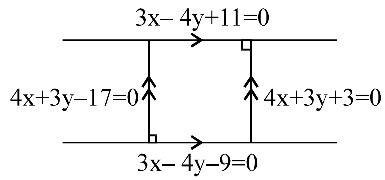
(R) a rhombus which is not a square.

(S) a square

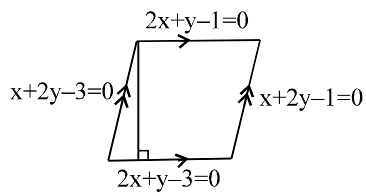
[Ans. (A) S; (B) R; (C) Q]

[Sol.

(A) $\left.\begin{array}{l} d_1 = \dfrac{20}{5} = 4 \\ d_2 = \dfrac{20}{5} = 4 \end{array}\right] \Rightarrow$ square



(B) $\left.\begin{array}{l} d_1 = \dfrac{2}{\sqrt{5}} \\ d_2 = \dfrac{2}{\sqrt{5}} \end{array}\right] \Rightarrow$ interior not 90° $\Rightarrow$ rhombus



(C) $2x^2 - 7xy + 3y^2 + 5x + 10y - 25 = 0 \equiv (x - 3y + 5)(2x - y - 5)$

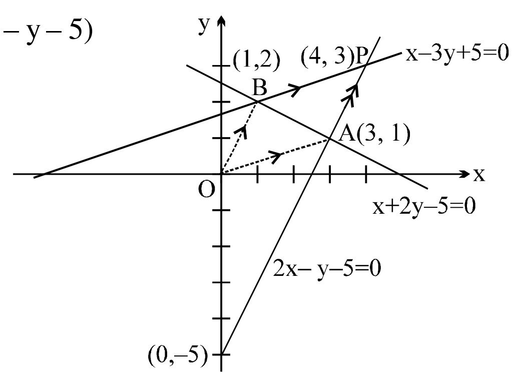
the point of intersection is (4, 3)

homogenising $f(x, y) = 0$ and $x + 2y - 5 = 0$

we get the homogeneous equation

$$2x^2 - 7xy + 3y^2 = 0$$

hence OAPB is a parallelogram]



Q.17 **Column-I** — **Column-II**

(A) If the straight line $y = kx \ \forall\ K \in I$ touches or passes outside the circle $x^2 + y^2 - 20y + 90 = 0$ then $|k|$ can have the value

(B) Two circles $x^2 + y^2 + px + py - 7 = 0$ and $x^2 + y^2 - 10x + 2py + 1 = 0$ intersect each other orthogonally then the value of p is

(C) If the equation $x^2 + y^2 + 2\lambda x + 4 = 0$ and $x^2 + y^2 - 4\lambda y + 8 = 0$ represent real circles then the value of λ can be

(D) Each side of a square is of length 4. The centre of the square is (3, 7). One diagonal of the square is parallel to $y = x$. The possible abscissae of the vertices of the square can be

(P) 1

(Q) 2

(R) 3

(S) 5

[Ans. (A) P, Q, R; (B) Q, R; (C) Q, R, S; (D) P, S]

[Sol. (A) $x^2 + k^2x^2 - 20kx + 90 = 0$

$x^2(1 + k^2) - 20kx + 90 = 0$

$D \le 0$

$400k^2 - 4 \times 90(1 + k^2) \le 0$

$10k^2 - 9 - 9k^2 \le 0$

$k^2 - 9 \le 0 \quad \Rightarrow \quad k \in [-3, 3]$

(B) $2\left(\frac{p}{2}\times 5 + \frac{p}{2}\times p\right) = -6 \Rightarrow -5p + p^2 + 6 = 0 \Rightarrow p^2 - 5p + 6 = 0 \Rightarrow p = 2$ or 3 **Ans.**

(C) $r_1^2 = \lambda^2 - 4 \ge 0$

$\lambda \in (-\infty, -2] \cup [2, \infty)$ ....(1)

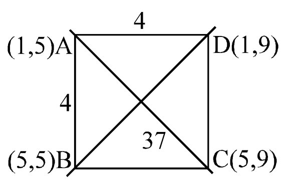
$r_2^2 = 4\lambda^2 - 8 \ge 0$

$\lambda^2 - 2 \ge 0$

$\lambda \in (-\infty, -\sqrt{2}\,] \cup [\sqrt{2}, \infty)$ ....(2)

(1) $\cap$ (2) is $\lambda \in (-\infty, -2] \cup [2, \infty)$ **Ans.**

(D)

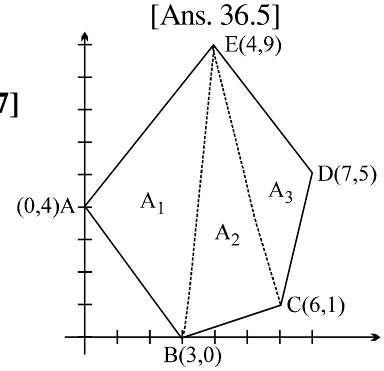


Ans. {1, 5}]

[SUBJECTIVE]

Q.18 Find the area of the pentagon whose vertices taken in order are (0, 4), (3, 0), (6, 1), (7, 5) and (4, 9). **[6]**

[Ans. 36.5]



[Sol. $A_1 = \frac{1}{2}\begin{vmatrix} 0 & 4 & 1 \\ 3 & 0 & 1 \\ 4 & 9 & 1 \end{vmatrix} = \frac{1}{2}\left|-4(-1)+1(27)\right| = \frac{31}{2}$ **[11th, 25-11-2007]**

$A_2 = \frac{1}{2}\begin{vmatrix} 3 & 0 & 1 \\ 4 & 9 & 1 \\ 6 & 1 & 1 \end{vmatrix} = \frac{1}{2}\left|3\cdot(9-1)+1\cdot(4-54)\right| = \frac{1}{2}\left|24-50\right| = 13$

$A_3 = \frac{1}{2}\begin{vmatrix} 4 & 9 & 1 \\ 7 & 5 & 1 \\ 6 & 1 & 1 \end{vmatrix} = \frac{1}{2}\left|4\times 4-9(1)+1(7-30)\right|$

$= \frac{1}{2}\left|16-9-23\right| = \frac{16}{2} = 8$

$\therefore$ Area of pentagon $= \frac{31}{2} + 13 + 8 = \frac{73}{2} = 36.5$ sq. units]